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Critical Investigation of Jauch's Approach to the Quantum Theory of Measurement

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To make Jauch's approach more realistic, his assumptions are modified in two ways: (1) On the quantum system plus the measuring apparatus (S+MA) after the measuring interaction has ceased, one can actually measure only operators of the form $A \otimes \sum_k b_k Q_k$, where A is any Hermitian operator for S, the resolution of the identity $\sum_k Q_k = 1$ defines MA as a classical system (following von Neumann), and the b_k are real numbers (S and MA are distant). (2) Measurement is defined in the most general way (including, besides first-kind, also second-kind and third-kind or indirect measurements). It is shown that Jauch's basic result that the microstates (statistical operators) of S+MA before and after the collapse correspond to the same macrostate (belong to the same equivalence class of microstates) remains valid under the above modifications, and that the significance of this result goes beyond measurement theory. On the other hand, it is argued that taking the orthodox (i.e. uncompromisingly quantum) view of quantum mechanics, it is not the collapse, but the Jauch-type macrostates that are spurious in a Jauch-type theory.

1. INTRODUCTION

To present the problem of the quantum theory of measurement and Jauch's attempt at a solution, we take the usual dynamical minimodel for first-kind measurement [essentially in Jauch's notation; cf. Jauch (1964, 1968)].

The state space $\mathscr{H}_{\rm I}$ of the quantum system I is two-dimensional, and we measure an observable that is given in spectral form as

$$A_{\rm I} = a_+|\varphi_+\rangle\langle\varphi_+| + a_-|\varphi_-\rangle\langle\varphi_-|, \qquad a_+ \neq a_-, \quad a_\pm \in R_1$$

The state space $\mathscr{H}_{\rm II}$ of the measuring apparatus (object II) is three-dimensional, and the measuring observable (the "pointer") is in spectral

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form

$$B_{\mathrm{II}} = b_0|\psi_0\rangle\langle\varphi_0| + b_+|\varphi_+\rangle\langle\psi_+| + b_-|\psi_-\rangle\langle\psi_-|$$

$$b_0 \neq b_+ \neq b_-, \qquad b_0 \neq b_-, \qquad b_0, b_\pm \in R_1$$

The evolution operator U of the composite system I+II (giving the dynamical basis of the measurement) is assumed to take the initial state $|\varphi_\pm\rangle|\psi_0\rangle$ with the sharp value $a_\pm$ of A_{I} into

$$U|\varphi_\pm\rangle|\psi_0\rangle = |\gamma\psi_\pm\rangle|\psi_\pm\rangle$$

If the object I is in a pure state that is a superposition

$$\alpha_+|\varphi_+\rangle + \alpha_-|\varphi_-\rangle, \qquad \alpha_+ \neq 0 \neq \alpha, \qquad |\alpha_+|^2 + |\alpha_-|^2 = 1$$

then, due to the linearity of the tensor product and of U, one obtains

$$U(\alpha_+|\varphi_+\rangle + \alpha_-|\varphi_-\rangle)|\psi_0\rangle = \alpha_+|\varphi_+\rangle|\psi_+\rangle + \alpha_-|\varphi_-\rangle|\psi_-\rangle \tag{1}$$

However, it is not the pure state (1), but the mixed state

$$|\alpha_+|^2|\varphi_+\rangle\langle\varphi_+|\otimes|\psi_+\rangle\langle\psi_+| + |\alpha_-|^2|\varphi_-\rangle\langle\varphi_-|\otimes|\psi_-\rangle\langle\psi_-| \tag{2}$$

that expresses the fact that in a fraction $|\alpha_\pm|^2$ of the ensemble of I+II systems the "pointer" takes the "position" $b_\pm$. How does the transition from (1) to (2), the so-called *collapse* [or process 1 in the original von Neumann terminology; cf. Chapter V in von Neumann (1955)] come about? This is *the problem* of the quantum theory of measurement.

As a contrast, let me state concisely the position that Niels Bohr took, as I understood it. In free formulation, his attitude can be given the form of two stipulations (Bohr 1949, 1963):

1. Every measurement has to be performed by using a *classical* measuring apparatus (MA).
2. The behavior of a classical MA during the measurement process should be described by classical physics. An attempt at a deeper quantum description of the MA and the interaction with it would be meaningless because human beings are doomed to understanding Nature via classical physics.

This attitude, accepted by the entire Copenhagen school of thought, represents an alternative to the above quantum theory leading to (1) [due to von Neumann, (1955)], and it does not lead to a problem. But Bohr's view actually fails to give insight into the way the measurement results come about, and they are the only statistical elements of quantum mechanics. Hence, this approach is unsatisfactory for many physicists (including, besides von Neumann and Jauch, the present author).

In von Neumann's alternative leading to (1) there is the well-known possibility of regression (another MA measures B_{II}, etc.), arriving at the ultimate observer: the human consciousness. According to von Neumann, it is the latter that is the ultimate source of the collapse (von Neumann, 1955).

Jauch finds von Neumann's conclusion unacceptable (Jauch, 1964, 1968). In his search for a loophole in von Neumann's theory, he takes resort to Bohr's stipulation 1, but without (2). In doing so he assumes that classical variables are actually compatible quantum mechanical observables.

According to Jauch, the statement that the quantum system II (the MA) is a *classical object* means that actually nothing but a set $\mathbf{O}^c$ of compatible observables can be measured on it.

At this point, Jauch makes an important observation concerning the general case when only a limited set $\mathbf{O}'$ of (not necessarily compatible) observables is actually measurable on a quantum system. Then, not every two statistical operators ρ and ρ' can be distinguished by measurement. In other words, an equivalence relation $\sim$ is introduced in the set $\mathbf{S}$ of all statistical operators:

$$\rho \sim \rho', \quad \rho, \rho' \in \mathbf{S}, \quad \text{if } \forall A \in \mathbf{O}': \quad \operatorname{Tr} A\rho = \operatorname{Tr} A\rho' \tag{3}$$

(If $\mathbf{O}'$ is replaced by $\mathbf{O}$, the set of all Hermitian operators, then $\sim$ in (3) becomes the equality.)

Subsequently, Jauch derives in a straightforward way the following result.

Theorem 1. The quotient set $\mathbf{S}/\sim$, in which the equivalence classes of statistical operators obtained via (3) are the elements, is a convex set, the convex combinations of which are performed through arbitrary class representatives.

This means that if $C_1, C_2, \ldots \in \mathbf{S}/\sim$, and $w_1 > 0$, $w_2 > 0, \ldots, \sum_i w_i = 1$, then also $\sum_i w_i C_i \in \mathbf{S}/\sim$, and taking arbitrarily $\rho_1 \in C_1$, $\rho_2 \in C_2, \ldots, \sum_i w_i C_i$ is the equivalence class to which $\sum_i w_i \rho_i$ belongs.

Jauch calls the classes *macrostates*, in contrast to their elements, which he calls *microstates*.

Let us return to the measurement problem. As mentioned, according to the Jauch definition of a classical object, on the MA (system II) only a set $\mathbf{O}^c$ of commuting Hermitian operators can be measured. It is now the leading idea of Jauch to obtain macrostates of I+II via (3) such that the microstates (1) and (2) are elements of the *same* macrostate.

Jauch takes for his $\mathbf{O}'$ set in application to I−II the observable

$$A_{\mathrm{I,II}} \equiv |\varphi_+\rangle\langle\varphi_+| \otimes |\psi_+\rangle\langle\psi_+| - |\varphi_-\rangle\langle\varphi_-| \otimes |\psi_-\rangle\langle\psi_-| \tag{4}$$

and all its functions, and he obtains the desired *basic result* that (1) and (2) belong to the same macrostate. Thus, Jauch concludes, the gap between

(1) and (2) is bridged, i.e., one has gained an understanding of how the collapse comes about: it turns out to be a *spurious effect.*

We are going to be concerned with three objections to Jauch's theory. The first comes from Jauch himself. At the end of his paper (Jauch, 1964), he makes the following remarks:

> In order to assert this result in full generality, the analysis carried through in this paper for a special case should be generalized and refined in several directions. One should extend it to an observable with more than two values, and one should also allow the possibility of a degenerate spectrum for the observed quantity. Then one should take into account that both systems I and II could be in a mixture before measurement begins. Furthermore, one should also include the case of continuous spectra. Finally, the discussion should then be extended to measurements of the second kind. Only then could we make these assertions in full generality.

The second and more important objection is leveled against the way in which Jauch introduces the classicalness of the MA in application to the microstates (1) and (2) of I+II, i.e., against the above set $\mathbf{O}'$ generated by (4). It discards in an unjustified way a number of important observables that certainly can be measured on I+II (e.g., all subsystem observables of I).

In the next section a physical argument is given on how to replace Jauch's incorrect Abelian set $\mathbf{O}'$ of actually measurable Hermitian operators on I+II by a correct non-Abelian set $\mathbf{O}\otimes\mathbf{O}^c(B_0)$. In Section 3 we define general measurement, classifying it into first-, second-, and third-kind measurements. In Section 4 it is proved that Jauch's basic result remains valid under the mentioned restriction to $\mathbf{O}\otimes\mathbf{O}^c(B_0)$ for general measurement.

In the last section the results of this study and Jauch's approach are critically examined, and the third objection against the Jauch theory is raised. Whether the theory is also capable of answering this challenge is not settle in this paper.

2. THE OPERATORS MEASURABLE ON THE SYSTEM-PLUS-APPARATUS AFTER THE MEASUREMENT

The set of all measurable operators on the quantum system plus MA (S+MA) after the measurement must contain all subsystem operators for S, because S is quantal, and it must not contain any subsystem operator B for the MA other than $B\in\mathbf{O}^c$, because the MA is classical. One wonders if there exists any reasonable set fulfilling these conditions that would give a less unrealistic Jauch-type theory. The answer is affirmative if one takes resort to distant correlations.

When the interaction between the quantum system I and the MA (object II) that leads to the measurement ceases, then subsystems I and II are separated and become distant systems. This means that they are far enough from each other so that not only can they no longer interact, but also that one can at best perform coincidence subsystem measurements on them [cf. the last section in Vujičić and Herbut (1984)]. Hence, **O**, the set of all Hermitian operators in $\mathscr{H}_{\mathrm{I}} \otimes \mathscr{H}_{\mathrm{II}}$, the state space of the composite system, is restricted to $\mathbf{O} \otimes \mathbf{O}^c$ containing all Hermitian operators of the form $A \otimes B$, where A is *any* Hermitian operator in $\mathscr{H}_{\mathrm{I}}$, and B is an element from some set $\boldsymbol{O}^c$ of commuting Hermitian operators characterizing the MA. The set $\mathbf{O} \otimes \mathbf{O}^c$ seems to be the correct set of all measurable operators in the final state of the S+MA system.

Let S be the set of all statistical operators ρ in $\mathscr{H}_{\mathrm{I}} \otimes \mathscr{H}_{\mathrm{II}}$. Jauch's procedure with $\mathbf{O} \otimes \mathbf{O}^c$ leads to the quantum macrostates that are elements of $\mathbf{S}/\sim$, where the equivalence relation $\sim$ is defined by $\mathbf{O} \otimes \mathbf{O}^c$ through (3).

Jauch does not specify his Abelian set $\mathbf{O}^c$ of measurable operators on the MA in more detail. In practice, one actually deals with a finite number of classical variables (coordinates and linear momenta, e.g.) and with their functions in a classical description. Therefore, it seems reasonable to assume that $\mathbf{O}^c$ also contains a finite number of commuting Hermitian operators and all their functions. Besides, the operators from $\mathbf{O}^c$ have pure discrete spectra due to the necessarily positive margins of error in measurements with classical instruments (von Neumann, 1955, p. 221). Von Neumann (1955, p. 174) shows that there exists a Hermitian operator, say B_0, with a pure discrete spectrum such that the above operators are its functions.

We call B_0 *the basic observable* of object II, and we give after *von Neumann*, the following *definition* of the fact that object II is *classical*: the set of all measurable Hermitian operators $\mathbf{O}^c(B_0)$ in $\mathscr{H}_{\mathrm{II}}$ consists of B_0 and of all its functions that are Hermitian operators.

Let the spectral form of the basic observable be

$$B_0 = \sum_{k \in K} b_k^0 Q_k, \qquad k \neq k' \Rightarrow b_k^0 \neq b_{k'}^0 \tag{5}$$

where K is an at most countabley infinite set, and Q_k are the eigenprojectors of B_0. The set K plays the role of an index set in (5), but essentially one has in mind the set of events $\{Q_k: k \in K\}$, each element of which is effectively an elementary event (atom) for the classical instrument (though the eigenvalues b^{0k} are, in general, degenerate).

Thus, defining

$$\mathbf{O}^c(B_0) \equiv \left\{ \sum_{k \in K} b_k Q_k \text{: Herm. op. } \sum_{k \in K} Q_k = 1 \text{ fixed} \right\} \tag{6}$$

we henceforth utilize von Neumann's above definition of a classical object.

Remark 1. Evidently, the Jauch method (3) of deriving macrostrates gives the same equivalence relation in **S** in $\mathscr{H}_{\text{II}}$ with $\mathbf{O}^c(B_0)$ defined by (6) as with $\{Q_k: k \in K\}$.

3. THE DIFFERENT KINDS OF QUANTUM MEASUREMENTS

If A_0 is any given Hermitian operator in $\mathscr{H}_{\text{I}}$, then, due to the spectral theorem, one has

$$A_0 = \int_{-\infty}^{+\infty} \lambda \, dE(\lambda)$$

where $E(\lambda)$ is the spectral measure (corresponding to A_0) of the interval $(-\infty, \lambda]$, and the rhs is a Stieltjes integral (Jauch, 1968; von Neumann, 1955). To evaluate it one can take a natural number n, break up the real axis

$$R_1 = \sum_{k=-\infty}^{\infty} \left(\frac{k-1}{n}, \frac{k}{n} \right]$$

select arbitrarily

$$\lambda_k \in \left(\frac{k-1}{n}, \frac{k}{n} \right]$$

then take two more natural numbers K and L and construct

$$A(n, K, L) \equiv \sum_{k=-k+1} \lambda_k E \left(\frac{k-1}{n}, \frac{k}{n} \right] \tag{7}$$

Then

$$\int_{-\infty}^{+\infty} \lambda \, dE(\lambda) = \lim A(n, K, L), \qquad n \to \infty, K \to \infty, L \to \infty$$

It is one of the tasks of the mathematics of the spectral theorem to prove that the threefold limit exists independently of the order of the limiting processes and independently of the arbitrary steps taken in constructing $A(n, K, L)$.

Any MA actually measures an observable like $A(n, K, L)$ that has a pure discrete and finite spectrum. Von Neumann's MA encompasses an infinite spectrum of A_0 in the general case (von Neumann, 1955, p. 220). This seems to be an unnecessarily gross idealization. It corresponds to an infinite double sequence $\{A(n, K, L): K \to \infty, L \to \infty\}$.

The *exact measurement* of $A(n, K, L)$ is an *approximate measurement* of A_0, and the approximation can be made arbitrarily good by selecting large enough n, K, L. If A_0 has a continuous spectrum, it is already taken care of by the universality of the spectral theorem. The separate points of the continuous spectrum cannot be measured anyway due to the positive margin of error of every measuring instrument (von Neumann, 1955, p. 221).

When a quantum state ρ_{I} is given and we want to measure A_0 in it with a certain precision, we choose $A(n, K, L)$ with n large enough to suit the desired margin of error, and K and L large enough so that the part of the spectrum of A_0 having nonzero probability in ρ_{I} is (approximately) incorporated in the interval $(-K/n, I/n]$ of $A(n, K, L)$ [cf. (7)]. In practice, the described choice of $A(n, K, L)$ amounts to the choice of a suitable MA (see below).

We drop n, K, L in $A(n, K, L)$, and we rewrite (7) as follows:

$$A = \sum_{m=1}^{M} a_m P_m, \quad m \neq m' \Rightarrow a_m \neq a'_m, \qquad \sum_{m=1}^{M} P_m = 1 \tag{8}$$

(if $a_m = 0$ appears in the spectrum of A, it is among the M terms).

In the state space $\mathscr{H}_{\mathrm{II}}$ of the MA a measuring observable (the "pointer") B_p is defined such that the spectrum of A and part of that of B_p are in a fixed correspondence (same index m):

$$B_p \doteq \sum_{m=1}^{M} b_m Q_m \tag{9}$$

all b_m distinct and nonzero. The eigenvalues b_m are the "positions" of the "pointer," and the occurrence of Q_m means that the "pointer" has "taken up the position" b_m, i.e., it "shows" the result a_m for A (in direct measurement; see below).

We are now equipped to define the three kinds of measurement. Let $\rho_{\mathrm{II}}^{(0)}$ be an initial (in general) mixed state of the MA. We assume that the MA has the initial "position" $b_{m=0} = 0$ in $\rho_{\mathrm{II}}^{(0)}$: $Q_{m=0}\rho_{\mathrm{II}}^{(0)} = \rho_{\mathrm{II}}^{(0)}$. Further, we assume that the quantum system and the MA are brought into contact (begin to interact) at some initial instant t_0, when the composite state is $\rho_{\mathrm{I}} \otimes \rho_{\mathrm{II}}^{(0)}$, $\leq_{\mathrm{I}}$ being an initial state of the quantum system. Let $U_{\mathrm{I,II}}(t - t_0) \equiv U$ be the evolution operator of the composite system that describes the interaction of I and II from t_0 to a later instant t when the measurement is completed. At this instant we have *the final state*

$$\rho = U(\rho_{\mathrm{I}} \otimes \rho_{\mathrm{II}}^{(0)}) U^{-1} \tag{10}$$

We are dealing with a *measurement of the first kind*, or a predictive measurement, or a repeatable one if the following two conditions are satisfied:

(i) The probability distribution of the measured observable A in the initial state ρ_{I} of the quantum system and of the "pointer" observable B_p

in the final composite state ρ determined by (10) coincide:

$$\mathrm{Tr}_{\mathrm{I}}\, \rho_{\mathrm{I}} P_m = \mathrm{Tr}_{\mathrm{I,II}}(1 \otimes Q_m)\rho, \qquad m = 1, 2, \ldots, M$$

(predictability).

(ii) The probability distribution of A in ρ_{I} and of A in the final state ρ are equal:

$$\mathrm{Tr}_{\mathrm{I}}\, \rho_{\mathrm{I}} P_m = \mathrm{Tr}_{\mathrm{I,II}}(P_m \otimes 1)\rho, \qquad m = 1, 2, \ldots, M$$

(repeatability).

If (i) is valid without (ii), then we have a *measurement of the second kind* or a retrodictive or a retrospective one.

Evidently, (i) implies that if A has a sharp value a_m in ρ_{I}, then the "pointer" B_p has the corresponding sharp value ("position") b_m in $_\rho$. This is why some early authors used the term predictability for requirement (i) (Landau and Peierls, 1931). In the same case if (ii) is also valid, then the sharp value of A remains sharp in ρ (this is why the measurement can be repeated and will give the same result). It is noteworthy that all this is meaningful for individual quantum systems and MAs.

To define another kind of measurement, let us consider the following requirement:

(iii) Let $v_m \equiv \mathrm{Tr}_{\mathrm{I}}\, \rho_{\mathrm{I}} P_m$, $m = 1, 2, \ldots, M$, be the initial probability distribution. We require that the final probabilities $w_m \equiv \mathrm{Tr}_{\mathrm{I,II}}\, \rho(1 \otimes Q_m)$ also form a probability distribution, i.e., that $\sum_{m=1}^{M} w_m = 1$. Further, we require that there exist a one-to-one map

$$X\colon \quad \{v_m\colon m = 1, 2, \ldots, M\} \to \{w_m\colon m = 1, 2, \ldots, M\}$$

that is *invertible.*

One should note that if (i) is valid, so is (iii), because then $w_m = v_m$, $m = 1, 2, \ldots, M$. If (i) and (ii) are not valid but (iii) is, then we have a *third kind* or indirect *measurement.* (First- and second-kind measurements are direct measurements.)

A subsequent measurement of B_p on ρ ("reading off" the results) should provide us with the w_m; the inverse map X^{-1} applied to these w_m allows us to infer the required v_m. [It was shown by Fine (1970) that map X boils down to a matrix characteristic of the MA.]

One should note that measurement of the third kind is an ensemble measurement that has no meaning for individual systems.

4. THE BASIC RESULT

The question that concerns us in this study, and to which Jauch's theory was addressed in the first place, is the problem of collapse. Any of the three kinds of measurement processes is not completed unless the collapse takes place, i.e., unless the final composite state ρ satisfies

$$\rho = \sum_{m=1}^{M} (1 \otimes Q_m)\rho(1 \otimes Q_m) \tag{11}$$

for every initial state ρ_{I} of the quantum system on which measurement can be performed with the given MA. It was shown by Fine (1970) that there is no evolution operator U that would lead to ρ satisfying (11) in either of the three kinds of measurement. (In Fine's proof the initial state $\rho_{\mathrm{II}}^{(0)}$ of the MA is a pure state.)

Returning to Jauch's approach, the relevant question is if the measuring observable B_p [cf. (9)] can be chosen in a reasonable way so that the lhs and the rhs of (11) belong to the same macrostate. (This is then tantamount to collapse in Jauch's theory.)

Definition. We specify the "pointer" observable B_p on the MA given by (9) to be more than a *function* of the basic observable $B_0 = \sum_{k \in K} b_k^0 Q_k$.

Theorem 2. Defining the classicalness of the MA in the manner of von Neumann, assuming that the composite state ρ of quantum system plus MA is distant (see Section 2 on both counts), and defining B_p as in the above Definition, one has that Jauch's basic result that ρ and $\sum_{m=1}^{M} (1 \otimes Q_m)\rho(1 \otimes Q_m)$ fall into the same equivalence class (macrostate) of S+MA *is valid* irrespective of the kind of measurement (first, second, or third) that via U (10) determines ρ.

Proof. According to Remark 1, it is sufficient to show that

$$\forall k \in K: \quad p_k' \equiv \mathrm{Tr}_{\mathrm{I,II}}(1 \otimes Q_k)\left[\sum_{m=1}^{M} (1 \otimes Q_m)\rho(1 \otimes Q_m)\right] = p_k$$

where

$$p_k \equiv \mathrm{Tr}_{\mathrm{I,II}}(1 \otimes Q_k)\rho$$

Due to the above Definition, one has $\forall k \in K$: $Q_k Q_m = \delta(k \in K_m) Q_k$, $m = 1, 2, \ldots, M$, where $K_m \equiv \{k\colon b_k^p = b_m\}$, and $\delta(k \in K_m) = 1$ if $k \in K_m$ and $= 0$ otherwise. This entails $\forall k \in K$: $p_k' = p_k$. ■

It is noteworthy that the collapse in Jauch's sense [i.e., the falling of ρ and of $\sum_{m=1}^{M} (1 \otimes Q_m)\rho(1 \otimes Q_m)$ into the same equivalence class of macrostate] is obtained from the definition of a distinct relation between S and MA in ρ (see Section 2). No details of the measuring process [entering through U in (10) that determines ρ] play any role in the proof. It is also irrelevant which particular coarsening $\sum_{k \in K} b_k^p Q_k$ of B_0 is the "pointer" observable. Hence, the significance of Theorem 2 goes beyond measurement theory.

Remark 2. If an object is defined as classical in the von Neumann manner, and a quantum system plus this object is in a distant microstate ρ, then, no matter how it has come into this state, the corresponding macrostate is a mixture of macrostates in each of which every classical variable b_k that is a coarsening of the basic variable b_k^0 has a *definite value.*

In view of Remark 2, one wonders what is the meaning of Theorem 2 for measurement? The directly measured probabilities $w_m \equiv \mathrm{Tr}_{\mathrm{I,II}}(1 \otimes Q_m)\rho$ have the required relation to $v_m \equiv \mathrm{Tr}_{\mathrm{I}}\, P_m \rho_{\mathrm{I}}$ (cf. Section 3) due to (10), i.e., due to the fact that the interaction between the quantum system and the MA establishes this relation between the w_m and the v_m. The Jauch-type result (Theorem 2) explains the definite "positions" b_m of the "pointer" B_p (and the corresponding definite values a_m of A in first-kind measurement) via decomposition using not the convexity of microstates in **S** (in $\mathcal{H}_{\mathrm{I}} \otimes \mathcal{H}_{\mathrm{II}}$), but the convexity that prevails in $\mathbf{S}/\sim$.

5. CONCLUDING REMARKS

1. One wonders if a Jauch-type theory of classical systems may have application also outside the quantum theory of measurement. First of all, there is the *preparation* of a quantum system in an initial state ρ_{I}. It is a cornerstone of quantum mechanics. There we have the final product ρ_{I} as a result of an interaction of a classical object (a laboratory arrangement, the preparator) with a quantum system.

Then, there is the *theory of molecules.* "A fundamental problem of theoretical chemistry is the discussion of the interaction between quantum systems and classical systems" (Primas, 1983, Abstract). Among other things, certain symmetry-breaking structures (chirality, isomerism, etc.) appear in molecular ground states that do not follow from the dynamical equations of the quantum system (the molecule), but can be described by the introduction of classical variables (Primas, 1983, Introduction). The concept of a Jauch-type macrostate may prove of some help in these problems.

2. There seem to exist two misconceptions in the literature about the concept of collapse.

(a) Some physicists seem to think that the "projection postulate" (the collaspse) is confined to minimal-disturbance, first-kind measurement (see Herbut, 1969, 1974), or even to von Neumann's original complete-observable measurement of this kind (von Neumann, 1955). In view of the fact that first-kind measurement is never typical in the laboratory, the problem of collapse then appears rather far-fetched, unnatural, and academic. Second-kind (individual system) measurement and third-kind (ensemble) measurement are typical in the laboratory, and the problem of collapse (into definite "pointer position" conposite states) is here just as unavoidable as in first-kind measurement. Hence, the problem of collapse is one of the very basic unsolved problems of quantum mechanics.

(b) One gets the impression in reading the work of some physicists that the authors have the illusion that one can speak of "statistical prediction" (which is the sole task of quantum mechanics, so one is told) without collapse. Even in a third-kind measurement, where the "pointer positions" b_m have no meaning for the individual I+II systems (only the distribution w_m gives indirect information on the measured observable A), one must *count* how many apparatuses give b_m in the ensemble to observe the relative frequency yielding w_m. And for this the collapse is a necessary logical premise.

3. One might say that most of the quantum theory of measurement writhes under the strain of the two famous antipodes: Bohr's quantum mechanically untouchable classical measuring instruments [see his stipulation (ii) in the Introduction], and von Neumann's human consciousness as the ultimate observer. One wonders if this is also so for Jauch's macrostate approach.

Jauch's approach with the von Neumann definition of a classical instrument taken in an absolute sense, i.e., if one believes that no observable outside $\mathbf{O}^c(B_0)$ can be measured on subsystem II *under any circumstances*, is a consistent solution of the problem of collapse; and, as such, it is intermediary between the antipodes. It reminds one of the theory of Daneri *et al.* (1962) (DLP) in this respect. They derive the collapse from quantum ergodic theory. The latter seems to be kind of a counterpart of the von Neumann definition of a classical object.

Incidentially, Bub has criticized the DLP work (Bub, 1968), and remarked that Bohr's position was consistent and did not require any derivation. Though true, this fact should not be held against the DLP theory, because in quantum measurement theory the aim is not to "cure" some inconsistency, but to replace one set of stipulations by another, a physically more plausible one. In this sense, DLP also seem to reject the second Bohr tenet (see the Introduction), keeping the first.

4. Finally, the fundamental question concerning Jauch's approach has to be raised. Does it actually solve the problem of collapse? If yes, what is our reply to the following paradox.

Let us return to the very simple example for the final state ρ given by the rhs of (1). Let us have in mind two observers: a quantum one, who can measure any observable on subsystem II, and a classical one, who can measure only a proper subset of commuting Hermitian operators $\mathbf{O}^c(B_0)$ on II (see Section 2). The collapse is the transition from (1) to the mixture (2) of some "pointer positions" b_+ and some of them b_-. Thus, the possibility of decomposing (1) is gained due to the restricted measuring capabilities of the classical observer. But does this make sense?! The composite state (1) is pure or homogeneous. Having in mind an ensemble of composite systems I + II, this means that the ensemble contains no two subensembles that would differ regarding *any* observable in $\mathscr{H}_{\mathrm{I}} \otimes \mathscr{H}_{\mathrm{II}}$. And then, restricting oneself to the operators from $\mathbf{O} \otimes \mathbf{O}^c(B_0) \subset \mathbf{O} \otimes \mathbf{O}$ (cf. Section 2), the homogeneous ensemble seems to become nevertheless decomposed into the definite "position" subensembles!

There are two possible replies.

(a) If we take the orthodox, i.e., uncompromisingly quantum, point of view, our answer must be: No, Jauch's approach does not solve the problem of collapse. The transition from (1) to (2) is only transformed away in it. It should not describe our laboratory experience because it is a spurious solution of a genuine problem.

(b) If we are prepared to compromise with an extra quantum mechanical idea, such as "the quantum observer in the above paradox does not exist in Nature" (cf. point 3 above), then the paradox disappears, and out answer must be: Yes, Jauch's approach definitely solves the spurious problem of collapse, or rather it disposes of the redundancy in $\mathscr{H}_{\mathrm{I}} \otimes \mathscr{H}_{\mathrm{II}}$ that leads to this apparent problem. This attitude appears to go from the orthodox point of view halfway toward Bohr's position.

A careful reading of Jauch's work (1964, 1968) reveals that he believed that if we looked hard enough for an observable of quantum system II not compatible with the ones from $\mathbf{O}^c(B_0)$, we would find one. Nevertheless, he did not seem prepared to go along with the conclusion under (a), if for no other reason then because we do see the definite "pointer positions" in our laboratory experience. This experimental fact suggests that Jauch's approach is in the right direction. But it has more methodological than foundational significance.

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